dual series equations in terms of the eigenfunctions of the Sturm-Liouville problem for a fourth order linear differential equation. Such dual equations are encountered when the method of homogeneous solutions is used to solve mixed problems.

REFERENCES

1. Aleksandrov, V. M., On solving a class of dual equations. Dokl. Akad. Nauk SSSR, Vol. 210, № 1, 1973.
2. Levitan, V. M. and Sargsian, I. S., Introduction to Spectral Theory. Moscow, "Nauka", 1970.
3. Marchenko, V. A., Spectral Theory of the Sturm-Liouville Operators. Kiev, "Naukova Dumka", 1972.
4. Babeshko, V. A., On the theory and applications of certain first kind integral equations. Dokl. Akad. Nauk SSSR, Vol. 204, № 2, 1972.
5. Tricomi, F., Differential Equations. N.Y., Wiley, 1957.
6. Babeshko, V. A., On an asymptotic method applicable to the solution of integral equations in the theory of elasticity and mathematical physics. PMM Vol. 30, № 4, 1966.
7. Aleksandrov, V. M., Asymptotic methods in contact problems of elasticity theory. PMM Vol. 32, № 4, 1968.
8. Aleksandrov, V. M. and Chebakov, M. I., On a method of solving dual integral equations. PMM Vol. 37, № 6, 1973.

Translated by L. K.

UDC 539.3

ON A METHOD OF SEPARATING THE STATE OF STRESS IN SHELLS OF NEGATIVE CURVATURE WITH ASYMPTOTIC EDGES

PMM Vol. 39, № 2, 1975, pp. 333-341

N. N. ROGACHEVA
(Moscow)

(Received September 13, 1973)

The method of separating the state of stress is the following for shells with non-asymptotic edges: the total state of stress of the shell for which all the conditions of applicability of membrane theory are satisfied, is separated into the fundamental state of stress and simple edge effects. The boundary conditions are hence also separated: the tangential conditions are satisfied because of arbitrariness of the membrane theory, and the nontangential conditions because of the simple edge effects.

The possibility is shown in this paper of using this method to analyze shells of negative curvature with four asymptotic edges. The theory of the generalized edge effect has been constructed in [1]. Here, the formulas of the generalized edge effect are derived by another method for convenience in the subsequent exposition. Boundary conditions are formulated for membrane theory and the generalized edge effect for diverse edge fixings.

All the terminology, notation, equations and relations of shell theory are borrowed from [1].

1. Let us construct a theory of the generalized edge effect in a first approximation. We select the curvilinear coordinates as follows: let the α-lines coincide with a family of asymptotic lines along one of which an asymptotic edge passes, and the β-lines are orthogonal.

The properties of the generalized edge effect can be clarified by expanding the desired quantities in asymptotic series, just as was done in [2]. This is associated with simple, but cumbersome reasoning. Omitting them, let us formulate the final results in the form of hypotheses.

1. Exactly as for the simple edge effect, the desired functions for the generalized edge effect vary rapidly in a direction orthogonal to the edge, while along the edge they vary considerably more slowly

$$\frac{1}{B}\frac{\partial W}{\partial \beta} \gg \frac{1}{A}\frac{\partial W}{\partial \alpha}, \qquad \frac{1}{B}\frac{\partial W}{\partial \beta} \gg W$$

Here W is any of the sought quantities, the displacement, force, or moments. We neglect the sought function itself and its first derivative with respect to α as compared with the derivative of the same function with respect to β.

2. The greatest of the displacements is w, normal to the middle surface. The tangential displacements u and v are related to w thus

$$w \sim \frac{1}{B}\frac{\partial u}{\partial \beta} \sim \frac{1}{B}\frac{\partial v}{\partial \beta} \tag{1.1}$$

The formulas (1.1) mean that w and the derivatives of u and v with respect to β are of the same order of magnitude.

3. The greatest of the tangential forces in magnitude will be T_1, the normal force acting in the section $\alpha = \text{const}$, while T_2 and S are related to T_1 as follows:

$$\frac{1}{B}\frac{\partial T_2}{\partial \beta} \sim T_1, \qquad \frac{1}{B}\frac{\partial S}{\partial \beta} \sim \frac{1}{A}\frac{\partial T_1}{\partial \alpha}$$

4. The elasticity relationships for the forces T_2 and S are replaced by the following formulas in the approximate theory of the generalized edge effect:

$$\varepsilon_2 + \nu\varepsilon_1 = 0, \qquad \omega = 0 \tag{1.2}$$

Using the hypotheses formulated, we can construct a theory of the generalized edge effect in a first approximation by proceeding exactly as in the construction of the theory of the simple edge effect in a first approximation, i.e. by keeping just the principal terms in each equation. In particular, exactly as in the theory of the simple egde effect, all the quantities except those desired can be considered as functions of just the variable α during differentiation with respect to β. Taking account of the first formula in (1.2), the elasticity relationship for T_1 is converted into the following:

$$T_1 = \frac{2Eh}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = 2Eh\varepsilon_1 \tag{1.3}$$

Expressing the strains in (1.2) in terms of the displacement and keeping the principal terms, we obtain

$$\frac{1}{B}\frac{\partial v}{\partial \beta} - \frac{1}{R_2'} w = 0, \qquad \frac{1}{B}\frac{\partial u}{\partial \beta} + \frac{2}{R_{12}} w = 0 \tag{1.4}$$

The largest of the bending strains is $\varkappa_2$; the component $\varkappa_1$ should be neglected as compared to $\varkappa_2$. We write the approximate formulas for the bending strains

$$\varkappa_2 = \frac{1}{B^2}\frac{\partial^2 w}{\partial \beta^2}, \quad \tau = \frac{1}{A}\frac{\partial}{\partial \alpha}\frac{1}{B}\frac{\partial w}{\partial \beta} \tag{1.5}$$

As follows from (1.5) and the appropriate elasticity relationships, the greatest of the moments is the bending moment G_2. The moment G_1 is a consequence of the Poisson effect. The moments G_1 and $H_1 = -H_2 = H$ are related to G_2 as follows:

$$G_1 = \nu G_2, \quad \frac{1}{B}\frac{\partial H}{\partial \beta} \sim \frac{1}{A}\frac{\partial G_2}{\partial \alpha}$$

The simplified elasticity relationships for the moments become

$$G_2 = -\frac{2Eh^3}{3(1-\nu^2)}\varkappa_2, \quad G_1 = -\frac{2Eh^3}{3(1-\nu^2)}\nu\varkappa_2, \quad H = \frac{2Eh^3}{3(1+\nu)}\tau \tag{1.6}$$

As is seen from the last two equilibrium equations, the larger of the transverse forces is N_2, related to N_1 as follows:

$$\frac{1}{A}\frac{\partial N_2}{\partial \alpha} \sim \frac{1}{B}\frac{\partial N_1}{\partial \beta}$$

We obtain the approximate equilibrium equations of the generalized edge effect by simplifying the equilibrium equations of shell theory [1]. The transverse forces are not essential in the first two equilibrium equations. We discard them; moreover, according to the first hypothesis, we neglect the shear force S in the first equation as compared with the derivative of the same force with respect to β and we discard the term $\partial BN_1/\partial\alpha$ as compared with $\partial AN_2/\partial\beta$ in the third. In exactly the same way, keeping the principal terms in the last two equilibrium equations, we arrive at the following equations

$$\frac{1}{A}\frac{\partial}{\partial \alpha}(BT_1) + \frac{\partial S}{\partial \beta} = 0, \quad \frac{\partial T_2}{\partial \beta} + \frac{1}{AB}\frac{\partial (B^2 S)}{\partial \alpha} - k_\alpha BT_1 = 0 \tag{1.7}$$

$$\frac{T_2}{R_2'} - \frac{2S}{R_{12}} + \frac{1}{B}\frac{\partial N_2}{\partial \beta} = 0, \quad N_2 = \frac{1}{B}\frac{\partial G_2}{\partial \beta}$$

$$N_1 = \frac{1}{AB}\frac{\partial}{\partial \alpha}(BG_1) - \frac{1}{B}\frac{\partial H}{\partial \beta} - k_\beta G_2 \tag{1.8}$$

$$k_\alpha = \frac{1}{AB}\frac{\partial A}{\partial \beta}, \quad k_\beta = \frac{1}{AB}\frac{\partial B}{\partial \alpha}, \quad S_1 = -S_2 = S, \quad 1/R_1' = 0$$

Let us convert (1.8). To do this, we substitute their expressions in terms of the displacement according to (1.5) and (1.6) for the moments in the right side, and we obtain

$$N_1 = -\frac{2Eh^3}{3(1-\nu^2)}\frac{1}{A}\frac{\partial}{\partial \alpha}\frac{1}{B^2}\frac{\partial^2 w}{\partial \beta^2}$$

or taking account of (1.5) and (1.6)

$$N_1 = \frac{1}{A}\frac{\partial G_2}{\partial \alpha} \tag{1.9}$$

Thus, a system of equations has been obtained for the nondegenerate edge effect in a first approximation in negative-curvature shells (1.3) – (1.7), (1.9). There are no elasticity relationships for T_2 and S. This means that these forces are static quantities to some accuracy, i.e. are determined from the equilibrium equations.

2. Using the second and third equilibrium equations in (1.7), the first equilibrium equation is converted into

$$\left(\frac{1}{A}\frac{\partial}{\partial\alpha}+\frac{k_\alpha R_{12}}{2R_2'}\right)BT_1+\frac{R_{12}}{2B^2}\frac{\partial^3 G_2}{\partial\beta^3}=0$$

Substituting their expressions in terms of the displacements in place of the forces in the equation obtained, then expressing w in terms of u, we obtain the governing equation of the generalized edge effect localized near the edge $\beta = \text{const}$, with respect to the displacement u

$$\left(\frac{1}{A}\frac{\partial}{\partial\alpha}+\frac{k_\alpha R_{12}}{2R_2'}\right)\left[B\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}-\frac{k_\alpha R_{12}}{2R_2'}u\right)\right]+\frac{h^2 R_{12}^2}{12(1-\nu^2)}\frac{1}{B^5}\frac{\partial^6 u}{\partial\beta^6}=0 \quad (2.1)$$

If the displacement u has been found, the remaining unknowns of shell theory can be expressed in terms of this quantity by using the following computational formulas:

$$v=-\frac{R_{12}}{2R_2'}u, \quad w=-\frac{R_{12}}{2}\frac{1}{B}\frac{\partial u}{\partial\beta} \quad (2.2)$$

$$T_1=2Eh\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}-k_\alpha\frac{R_{12}}{2R_2'}u\right)$$

$$\frac{\partial^2 T_2}{\partial\beta^2}=2Eh\left(\frac{1}{AB}\frac{\partial}{\partial\alpha}\frac{B^2}{A}\frac{\partial}{\partial\alpha}B+k_\alpha B\frac{\partial}{\partial\beta}\right)\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}-\frac{k_\alpha R_{12}}{2R_2'}u\right)$$

$$\frac{\partial S}{\partial\beta}=-\frac{2Eh}{A}\frac{\partial}{\partial\alpha}\left[B\left(\frac{1}{A}\frac{\partial u}{\partial\alpha}-k_\alpha\frac{R_{12}}{2R_2'}u\right)\right]$$

$$G_1=\nu\frac{Eh^3}{3(1-\nu^2)}\frac{R_{12}}{B^3}\frac{\partial^3 u}{\partial\beta^3}, \quad G_2=\frac{Eh^3}{3(1-\nu^2)}\frac{R_{12}}{B^3}\frac{\partial^3 u}{\partial\beta^3}$$

$$H=-\frac{Eh^3}{3(1+\nu)}\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{R_{12}}{B^2}\frac{\partial^2 u}{\partial\beta^2}\right)$$

$$N_1=\frac{Eh^3}{3(1-\nu^2)}\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{R_{12}}{B^3}\frac{\partial^3 u}{\partial\beta^3}\right), \quad N_2=\frac{Eh^3}{3(1-\nu^2)}\frac{R_{12}}{B^4}\frac{\partial^4 u}{\partial\beta^4}$$

Subsequently we shall need a relationship connecting the forces S_2, T_2 and N_2. To do this, let us convert the second equation in (1.7) by using the first and third. Consequently, we have

$$\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{k_\alpha}T_2\right)+\frac{1}{A}\frac{\partial}{\partial\alpha}\left[\frac{1}{ABk_\alpha}\frac{\partial}{\partial\alpha}\left(\frac{BR_{12}}{2}N_2\right)\right]-S_2=0 \quad (2.3)$$

3. All the coefficients in (2.1) can be considered as functions of just α, hence, the governing equation of the generalized edge effect can be integrated by separation of variables. Because of the separation of variables we obtain a second-order equation in α with variable coefficients which should generally be integrated by numerical methods, and a sixth-order equation in β with constant coefficients.

The governing equation can be integrated for some surfaces without relying on numerical methods. Let us examine these particular cases in greater detail: (1) the shell middle surface is a minimal surface; (2) the asymptotic α-lines near the edge $\beta = \text{const}$ are simultaneously geodesics.

The coordinate lines coinciding with the asymptotic lines are orthogonal for a minimal surface. Hence, in the case (1) the normal curvatures of the α- and β-lines equal zero

$$1/R_1' = 1/R_2' = 0 \quad (3.1)$$

In the case (2) the geodesic curvature of the asymptotic α-lines is zero near the edge

$$k_\alpha = 0, \quad 1 / R_1' = 0 \tag{3.2}$$

moreover

$$1 / R_{12} \neq 0$$

(since it is considered that the Gaussian surface curvature is everywhere nonzero).

Let us write the first Codazzi equation

$$\frac{\partial}{\partial \beta}\left(\frac{A}{R_1'}\right) + \frac{1}{B}\frac{\partial}{\partial \alpha}\left(\frac{B^2}{R_{12}}\right) - k_\alpha \frac{AB}{R_2'} = 0$$

Because of (3.1) and (3.2) this equation is simplified for the particular cases under consideration

$$\frac{\partial}{\partial \alpha}\left(\frac{B^2}{R_{12}}\right) = 0$$

and therefore, B^2 / R_{12} is a function of just β. Moreover, since the coefficients of the first quadratic form can be considered functions of just α near the edge, we then perform the following change of variables:

$$\frac{B}{A}\frac{\partial}{\partial \alpha} = \frac{\partial}{\partial \xi}, \quad \frac{1}{\sqrt[3]{12(1-\nu^2)}}\sqrt[3]{\frac{R_{12}h}{B^2}}\frac{\partial}{\partial \beta} = \frac{\partial}{\partial \eta}$$

The edge $\beta = \mathrm{const}$ consequently, goes over into the line $\eta = \mathrm{const}$, and (2.1) is reduced to

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^6 u}{\partial \eta^6} = 0 \tag{3.3}$$

Using the method of separation of variables, we find the solution of (3.3) which decreases with distance from the edges

$$\begin{aligned} u = {} & \{\sin c_n \xi \, [A_1 e^{k(\eta-\eta_1)} + A_2 \sin \sqrt{3}\, k(\eta-\eta_1) + \\ & A_3 \cos \sqrt{3}\, k(\eta-\eta_1)] + \cos c_n \xi \, [A_4 e^{k(\eta-\eta_1)} + \\ & A_5 \sin \sqrt{3}\, k(\eta-\eta_1) + A_6 \cos \sqrt{3}\, k(\eta-\eta_1)]\} e^{k(\eta-\eta_1)} + \\ & \{\sin c_n \xi [B_1 e^{-k(\eta-\eta_2)} + B_2 \sin \sqrt{3}\, k(\eta-\eta_2) + \\ & B_3 \cos \sqrt{3}\, k(\eta-\eta_2)] + \cos c_n \xi \, [B_4 e^{-k(\eta-\eta_2)} + \\ & B_5 \sin \sqrt{3}\, k(\eta-\eta_2) + B_6 \cos \sqrt{3}\, k(\eta-\eta_2)]\} e^{-k(\eta-\eta_2)}, \; k = \sqrt[3]{c_n}/2 \end{aligned} \tag{3.4}$$

The part of the solution included in the first braces is the edge effect near the edge $\eta = \eta_1$ in the domain $\eta \leqslant \eta_1$. The solution describing the edge effect at the edge $\eta = \eta_2$ for $\eta \geqslant \eta_2$ is in the second braces. If the distance between the edges is sufficiently great, so that the edge effect originating near one edge is damped out succsessfully before the other, then the mutual influence of the edges can be neglected and only the first part of the solution need be kept in (3.4) in a computation of the edge effect at the edge $\eta = \eta_1$ and only the second part near the edge $\eta = \eta_2$. Hence, the boundary conditions on each edge can be satisfied separately.

Let ξ vary between the limits $(-l, l)$ along the edge $\eta = \eta_1$ (this can always be achieved by selecting the origin of the variable ξ). Selecting the constant c_n in (3.4) as $c_n = \pi n/l$, $n = 0, 1, 2, \ldots$, we obtain the solution of (3.3) as a trigonometric series whose n-th term is defined by (3.4).

As is seen from the structure of (3.3), three boundary conditions can be imposed on its integration at each edge $\eta = \mathrm{const}$. The functions in the right sides can hence be expanded in trigonometric series. By satisfying the boundary conditions we determine the constants A_i and B_i $(i = 1, 2, \ldots, 6)$.

4. Let us apply the theory of the generalized edge effect derived in Sects. 1 and 2 for a computation of negative-curvature shells with asymptotic by the method of separation of the state of stress.

In computing the state of stress by this method, the need arises to separate the boundary conditions into boundary conditions for membrane theory and conditions for the edge effects.

When the shell edges are not asymptotic (cf. [1]) the membrane equations are integrated by satisfying two boundary conditions at each point of the edge. The residuals which hence appear in the nontangential boundary conditions are eliminated by using the simple edge effect. Consequently, secondary residuals are obtained in the tangential conditions, which turn out to be small. Examples of clamped and hinge-supported edges easily show that the smallness of the secondary residuals is assured by the fact that the tangential displacements in the simple edge effect are considerably less than the normal displacements.

An asymptotic line on a negative-curvature surface is a double characteristic for the membrane equations, hence, only one boundary condition on each edge can be satisfied by membrane theory for negative-curvature shells with asymptotic edges. The residuals obtained in the remaining three boundary conditions can be reduced by using the generalized effect (see Sect. 3).

The boundary condition for membrane theory must indeed be selected in this case so that the secondary residuals, appearing because of the computation by the method of separation. A difficulty hence originates which is associated with the fact that only one of the two tangential boundary conditions for membrane theory must be conserved. By analogy with the scheme described above for the application of the method of separation to shells with nonasymptotic edges, we assume that that tangential condition must be kept in which the appropriate displacement or force in the theory of the generalized edge effect turns out to be least (it can be confirmed by direct substitution that the secondary residuals hence actually turn out to be small).

Let us examine particular cases.

Clamped edge. The boundary conditions on the edge $\beta = \beta_0$ are

$$u = v = w = \gamma_2 = 0$$

Here the first two boundary conditions are tangential. Without altering the substance of the problem, various linear combinations of these can be made. It turns out that the single boundary condition for membrane theory is

$$v + \frac{R_{12}}{2R_2'} u = 0, \quad \beta = \beta_0 \tag{4.1}$$

since the appropriate tangential displacements in the generalized edge effect vanish in a first approximation because of the first equality in (2.2).

For minimal surfaces, the curvature of the β-line equals zero $(1 / R'_2 = 0)$ at the edge $\beta = \beta_0$ and the condition for membrane theory is simplified

$$v = 0, \quad \beta = \beta_0 \tag{4.2}$$

Simply supported edge. The boundary conditions on the edge $\beta = \beta_0$ are

$$T_2 = S_2 = G_2 = N_2 = 0 \qquad (4.3)$$

The first two conditions in (4.3) are tangential. The following combination can be made from them:

$$\frac{1}{A}\frac{\partial}{\partial \alpha}\left(\frac{1}{k_\alpha}T_2\right) - S_2 = 0 \qquad (4.4)$$

which must indeed be taken as the boundary condition on the edge $\beta = \beta_0$ in membrane theory. In fact, the transverse force N_2 in membrane theory is small compared to the tangential forces T_2 and S_2, hence, the left side of (4.4) differs slightly from the left side of (2.3), and this latter is approximately zero in the edge effect.

In the particular case when the edge line is a geodesic (for example the edge is rectilinear), the force T_2 is essentially greater than the force S_2 in the edge effect (see the second equation in (1.7)), and

$$T_2 = 0, \quad \beta = \beta_0 \qquad (4.5)$$

should be taken in place of condition (4.4).

H i n g e - s u p p o r t e d e d g e. The boundary conditions are

$$T_2 = u = w = G_2 = 0, \quad \beta = \beta_0$$

Of the two tangential boundary conditions for membrane theory,

$$T_2 = 0, \quad \beta = \beta_0 \qquad (4.6)$$

should be selected since, as is seen from (2.2), T_2 is less than $2Ehu$ in the theory of the edge effect.

The boundary conditions (4.2), (4.5), (4.6) have been obtained earlier in [3]. Particular kinds of negative-curvature shells had hence been examined, for which these conditions are actually satisfied.

It should be kept in mind that orthogonal coordinates were selected to construct the approximate theory of the generalized edge effect. At the same time, conjugate non-orthogonal coordinates ρ and μ coincident with both families of asymptotic lines are more convenient for membrane theory. Henceforth, we shall use either coordinates by keeping in mind that the edge line is given by identical equations $\rho = \text{const}$, $\alpha = \text{const}$ or $\mu = \text{const}$, $\beta = \text{const}$, in both systems.

However, the form of the boundary conditions depends on the coordinates selected. Without examing the passage from one coordinate system to the other, let us write the boundary conditions in the conjugate (ρ, μ) coordinates on the edge $\mu = \text{const}$

$$u_\mu + 2\, u_\rho \cos\chi = 0 \quad [(4.1)] \qquad (4.7)$$

$$T_\mu = 0 \quad [(4.5),\ (4.6)] \qquad (4.8)$$

$$\frac{1}{A_\rho}\frac{\partial}{\partial \rho}\left[\left(\frac{1}{A_\rho}\frac{\partial \chi}{\partial \rho} - k_\mu \operatorname{ctg}\chi + \frac{k_\rho}{\sin\chi}\right)^{-1} T_\mu \sin\chi\right] + T_\mu\cos\chi - S_\mu = 0 \ [(4.4)] \qquad (4.9)$$

where χ is the angle between the coordinate lines. The number of the corresponding boundary conditions in orthogonal coordinates is on the right within brackets. The subscripts ρ and μ mean that the quantities refer to the conjugate coordinates.

5. Let us discuss under which conditions will the edge problems of membrane theory be correct. For simplicity in the discussion, let us consider a negative-curvature shell

bounded by rectilinear asymptotic lines (in this case the membrane solution is found by using quadratures [4]). The deductions obtained in this particular case are general in nature.

The equilibrium equations in the conjugate coordinate system can briefly be written as

$$\frac{\partial}{\partial \rho}(A_\mu T_\rho) = F_1(X, Y, Z),\ \frac{\partial}{\partial \mu}(A_\rho T_\mu) = F_2(X, Y, Z),\ S_{\rho\mu} = F(Z) \quad (5.1)$$

Known functions of the components of the external loads X, Y, Z are in the right side of (5.1). After simple manipulations, we obtain differential equations for the displacement from the elasticity relationships (5.2)

$$\frac{\partial}{\partial \rho}(u_\rho + u_\mu \cos\chi) = f_1(T_\rho, T_\mu, S_{\rho\mu}) \quad \frac{\partial}{\partial \rho}(u_\mu + u_\rho \cos\chi) = f_2(T_\rho, T_\mu, S_{\rho\mu})$$

Here f_1 and f_2 are known functions of the forces.

We examine various combinations of the boundary conditions presented in Sect. 4.

Let the two adjacent edges be clamped, and the other two either hinged or simply supported. This means that conditions on the tangential displacements of the type (4.7) are given on the clamped edges $\rho = \rho_0$, $\mu = \mu_0$ and static conditions of the form (4.8), (4.9) on the edges $\rho = \rho_1$, $\mu = \mu_1$. In this case the problem is divided into two stages. The first stage is the solution of the static problem. Two arbitrary functions, which appear because of integrating the system (5.1) are determined from the static boundary conditions. The second stage is the solution of the geometric problem (5.2), whose arbitrariness is determined from the conditions on the edges $\rho = \rho_0$, $\mu = \mu_0$. Under such conditions the problem is correct. If three edges are clamped, or the shell is clamped over the whole edge outline, then, as is easy to see from the solution of (5.1), (5.2), all the arbitrary functions of integration are determined from the boundary conditions.

The membrane theory will be incorrect for all other clampings.

For example, let us examine a shell clamped along the edge $\rho = \rho_0$ and simply (or hinge) supported on the three remaining edges. In this case we have three static conditions of the type (4.9), (4.8) on the edges $\rho = \rho_1$, $\mu = \mu_1$, $\mu = \mu_0$ and one geometric condition (4.7) on the edge $\rho = \rho_0$. Integrating Eq. (5.1) and satisfying the static condition on the edges $\rho = \rho_1$ and $\mu = \mu_0$, we obtain an unavoidable discrepancy on the edge $\mu = \mu_1$. In general, one static condition cannot therefore be satisfied. At the same time, one condition is not sufficient to determine the displacement on the edge $\rho = \rho_0$ which signifies incorrectness of the problem.

The same cases were also obtained when applying the method of separation to shells with nonasymptotic edges. It has been shown in [5] that methods exist for eliminating the incorrectness. It is also possible to eliminate the incorrectness in the case presented, but we shall not examine this question.

Thus for the problem to be correct in the membrane formulation, not less than two adjacent edges must be clamped on the shell.

The author is grateful to A. L. Gol'denveizer for constant attention to the research and for valuable remarks.

REFERENCES

1. Gol'denveizer, A. L., Theory of Elastic Thin Shells. (English translation), Pergamon Press, Book № 09561, 1961.

2. Gol'denveizer, A. L., On the two-dimensional equations of a linear theory

of thin elastic shells. In: Problems of Hydrodynamics and Mechanics of Continuous Media. "Nauka", Moscow, 1969.
3. Visarion, V. and Stănescu, C., Calculul stărilor de tensiune în teoria placilor curbe. Bucuresti Acad. Republicii Socialiste. Romania, 1969.
4. Rabotnov, Iu. N., Some solutions of the membrane theory of shells. PMM Vol. 10, №5, 6, 1946.
5. Gol'denveizer, A. L. and Zveriaev, E. M., State of stress in unconstrained shells of zero curvature. PMM Vol. 35, №2, 1971.

Translated by M. D. F.

UDC 539.3:534.1

ASYMPTOTIC METHOD OF INVESTIGATION OF SHORT-WAVE OSCILLATIONS OF SHELLS

PMM Vol. 39, №2, 1975, pp. 342-351
G. N. CHERNYSHEV
(Moscow)
(Received May 14, 1974)

Short-wave oscillations of shells located in a certain sufficiently narrow boundary region, are investigated. An asymptotic integration procedure is constructed, by analogy with the methods presented in papers [1, 2]. Attention is paid mainly to the natural oscillations of shells, but forced oscillations are also considered at the end of this paper. The region of the oscillations here investigated is arbitrarily divided in two parts: one low frequency and the other high frequency. The equation of the first approximation for high frequency oscillations is the simplest, therefore this equation is considered first of all and the asymptotic procedure of integration is constructed; afterwards this method is generalized for low frequency short-wave oscillations of shells.

1. In this paper the oscillations are considered to be short-wave, if they are defined by the equations of a rapidly varying state of stress. Moreover, the so-called quasitransverse oscillations are considered when in the equations the inertial terms relating to tangential displacements, are discarded. With these assumptions the equations are written in the following form (using here the notation from the monograph [3]):

$$h_1^2 \Delta^2 w - E^{-1} h^{-1} \Delta_1 c - \lambda^2 w = 0, \quad \lambda^2 = \rho E^{-1} \omega^2 Eh \Delta_1 w + \qquad (1.1)$$
$$\Delta^2 c = 0, \quad h_1^2 = h^2 [3(1-\sigma^2)]^{-1}, \quad \Delta = B^{-1}\partial_\alpha (B\partial_\alpha) +$$
$$A^{-1}\partial_\beta (A\partial_\beta), \quad \partial_\alpha = A^{-1}\partial / \partial\alpha, \quad \partial_\beta = B^{-1}\partial / \partial\beta$$
$$\Delta_1 = B^{-1}\partial_\alpha (BR_2^{-1}\partial_\alpha) + A^{-1}\partial_\beta (AR_1^{-1}\partial_\beta)$$

It is assumed that the system of coordinates for the middle surface is referred to the principal lines of curvatures and the boundary is represented as a smooth convex line without corners (convexity condition will be considered later); the middle surface must be sufficiently smooth. Oscillations with frequencies satisfying the inequality

$$\lambda \gg \max (R_1^{-1}, R_2^{-1}) \qquad (1.2)$$